LIMITS REVIEW

- 1. For "well-behaved" functions Direct Substitution: Example: $\lim_{x \to -3} (3x+2) = (3(-3)+2) = -7$
- 2. For "Not-So-Well-Behaved" functions Remember to AVOID $\frac{0}{0}$
 - a. Try to simplify first:

$$\lim_{x \to 0} \frac{\left[\frac{1}{x+4} - \frac{1}{4}\right]}{x} \\
\lim_{x \to 0} \frac{\left[\frac{1}{x+4} - \frac{1}{4}\right]}{x} \cdot \frac{4(x+4)}{4(x+4)} \\
\lim_{x \to 0} \frac{4 - (x+4)}{4x(x+4)} \\
\lim_{x \to 0} \frac{-x}{4x(x+4)} \\
\lim_{x \to 0} \frac{-1}{4(x+4)} \\
\lim_{x \to 0} \frac{-1}{4(x+4)} \\
-\frac{1}{16}$$

b. Try to factor and cancel:

$$\lim_{x \to 2} \frac{2-x}{x^2 - 4} \\
\lim_{x \to 2} \frac{2-x}{(x+2)(x-2)} \\
\lim_{x \to 2} \frac{-1}{x+2} \\
-\frac{1}{4}$$

***Recall:

x = 2 is a removable discontinuity x = -2 is a non-removable discontinuity

c. Try to rationalize the numerator:

$$\lim_{x \to 3} \frac{(\sqrt{x+1}-2)}{x-3}$$

$$\lim_{x \to 3} \frac{(\sqrt{x+1}-2)}{x-3} \cdot \frac{(\sqrt{x+1}+2)}{(\sqrt{x+1}+2)}$$

$$\lim_{x \to 3} \frac{x+1-4}{(x-3)(\sqrt{x+1}+2)}$$

$$\lim_{x \to 3} \frac{x-3}{(x-3)(\sqrt{x+1}+2)}$$

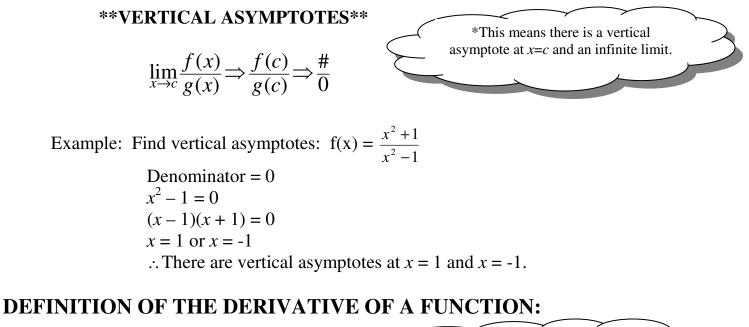
$$\lim_{x \to 3} \frac{1}{(\sqrt{x+1}+2)}$$

$$\frac{1}{4}$$

INFINITE LIMITS:

 $\lim_{x\to c} = \pm \infty$

This does not mean that the limit exists. But it does tell you how the limit fails to exist and denotes the unbounded behavior of the function.





LIMITS AT INFINITY:

 $\lim_{x\to\infty}\frac{c}{x^r}=0$

Avoid $\frac{\infty}{\infty}$ by dividing through by the highest power of x in the denominator.

****HORIZONTAL ASYMPTOTES****

If $\lim_{x \to \pm \infty} f(x) = L$, y = L is a horizontal asymptote.

L'HOPITAL'S RULE:

Only works for indeterminate forms: $\frac{0}{0}, \frac{\infty}{\infty}, \frac{\infty}{0}, \frac{0}{\infty}$

 $\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$